Section 2-7

Mathematical Models: Constructing Models

1.) In economics, revenue R, in dollars, is defined as the amount of money received from the sale of a product and is equal to the unit selling price p, in dollars, of the product times the number x of units actually sold. That is,

$$R = xp$$

In economics, the Law of Demand states that p and x are related. As one increases, the other decreases. The price p, in dollars, and the quantity x sold of a certain product obey the demand equation

$$p = \frac{-1}{3}x + 100, \quad 0 \leq x \leq 300$$

a.) Express the revenue R as a function of x.

b.) What is the revenue if 100 units are sold?

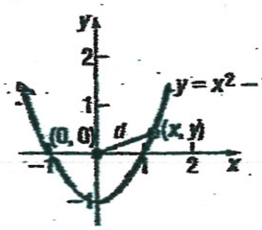
c.) Graph the revenue function using a graphing calculator.

d.) What quantity x maximizes revenue? What is the maximum revenue?

e.) What price should the company charge to maximize revenue?

2.) The perimeter of a rectangle is 50 feet. Express its area A as a function of the length, l, of a side.

3.) Let P(x, y) be a point of the graph of $y = x^2 - 1$.



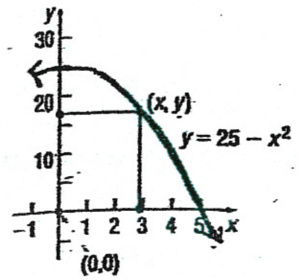
a.) Express the distance d from P to the origin as a function of x.

b.) What is d if $x = 0$? If $x = 1$?

c.) What is d if $x = \frac{\sqrt{2}}{2}$?

d.) Find the points on the graph closest to the origin.

4.) A rectangle has one corner on the graph of $y = 25 - x^2$, another at the origin, a third on the positive y-axis, and the fourth on the positive x-axis.



a.) Express the area A of the rectangle as a function of x.

b.) What is the domain of A?

c.) Graph $A = A(x)$.

d.) For what value of x is the area the largest?

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equation $$p = \frac{-1}{3}x + 100, \quad 0 \le x \le 300$$

a.) Express the revenue R as a function of x.

$R = x\left(-\frac{1}{3}x + 100\right) \rightarrow R = -\frac{1}{3}x^2 + 100x$

b.) What is the revenue if 100 units are sold?

$R = 100\left[-\frac{1}{3}(100) + 100\right]$ or $R = -\frac{1}{3}(100)^2 + 100(100) = \boxed{\$6666.67}$

c.) Graph the revenue function using a graphing calculator.

Xmin −10 XMAX 500 XSCL 100 | Ymin −500 YMAX 10000 YSCL 500

d.) What quantity x maximizes revenue? What is the maximum revenue?

$\boxed{x = 150}$; $R = -\frac{1}{3}(150)^2 + 100(150) = \boxed{\$7500}$

e.) What price should the company charge to maximize revenue?

$p = -\frac{1}{3}(150) + 100 \rightarrow \boxed{\$50}$

2.) The perimeter of a rectangle is 50 feet. Express its area A as a function of the length, l, of a side.

$P = 2L + 2W$

$\frac{50}{2} = \frac{\cancel{2}(L+W)}{\cancel{2}}$

$25 = L + W$

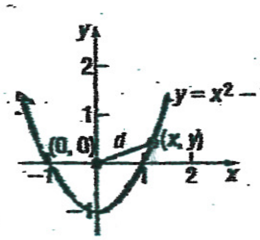
$25 - L = W$

$A = LW$

$A = L(25 - L)$

$\boxed{A = 25L - L^2}$

3.) Let P(x, y) be a point of the graph of $y = x^2 - 1$.



a.) Express the distance d from P to the origin as a function of x.

$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x - 0)^2 + (x^2 - 1 - 0)^2}$

$D = \sqrt{x^2 + x^4 - 2x^2 + 1} = \boxed{\sqrt{x^4 - x^2 + 1} = D}$

b.) What is d if x = 0? If x = 1?

If $x = 0$, $D = \sqrt{1} = \boxed{1}$; If $x = 1$, $D = \sqrt{(1)^4 - (1)^2 + 1}$

$D = \sqrt{1} = \boxed{1}$

c.) What is d if $x = \frac{\sqrt{2}}{2}$?

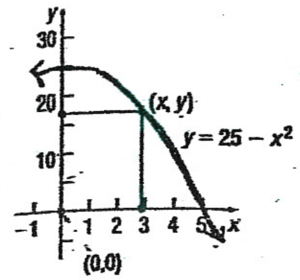
$D = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^4 - \left(\frac{\sqrt{2}}{2}\right)^2 + 1} = \sqrt{\frac{1}{4} - \frac{1}{2} + 1} = \sqrt{\frac{3}{4}} = \boxed{\frac{\sqrt{3}}{2}}$

d.) Find the points on the graph closest to the origin.

$(0)^2 = \left(\sqrt{x^4 - x^2 + 1}\right)^2 \rightarrow 0 = x^4 - x^2 + 1$ (put into calc; graph & find mins

Zoom 6, then Zoom In a little at the min values, 2nd trace, min (find both) (−0.7071, 0.75) (0.7071, 0.75)

4.) A rectangle has one corner on the graph of $y = 25 - x^2$, another at the origin, a third on the positive y-axis, and the fourth on the positive x-axis.



a.) Express the area A of the rectangle as a function of x.

$A = x(25 - x^2) \rightarrow \boxed{A = 25x - x^3}$

b.) What is the domain of A?

$0 < x < 5$ [starts at $x = 0$ + crosses y-axis at $x = 5$]

c.) Graph $A = A(x)$.

d.) For what value of x is the area the largest?

2ND trace → max → L bound, R bound for max

$\boxed{x \approx 2.887}$; $A = 48.11$